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Supersymmetric and κ-invariant coincident D0-branes

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Abstract: We propose a generic supersymmetric and κ-invariant action for describing coincident D0-branes with non-abelian matter fields on their worldline. The action is shown to be in agreement with the Matrix Theory limit of the ND0-brane effective action.

Keywords: D-branes, M(atrix) Theories, Space-Time Symmetries, Supersymmetric Effective Theories.

Contents

1. Introduction

The construction of a complete action for the supersymmetric non-abelian Dirac-Born-Infeld theory which could be associated with the effective action of the open superstring theory, and the construction of corresponding supersymmetric actions for coincident Dirichlet branes is still lacking. At the bosonic level the problem is that though one knows the full structure of all the non-abelian terms which enter the effective action under the symmetrized trace in a DBI manner [1, 2, 3], a generic recipe of constructing the commutator and higher derivative terms [4] has not been found yet. Also adding fermionic terms to the symmetrized trace part of the non-abelian DBI action for coincident branes via supersymmetrization encounters difficulties. By now the supersymmetric and κ-invariant actions with the symmetrized trace have been constructed only for N coincident D0-branes (ND0-branes) in $N = 1$, $D = 2$ [5], with partial results obtained also for super ND0-branes in a space-time of arbitrary dimension [6], and for space filling ND2-branes in $N = 2$, $D = 3$ superspace [7]. The space filling supersymmetric ND2-brane model of [7] has been constructed with the use of superembedding methods (see [8] for a review), while the super ND0-brane models have been obtained by more traditional methods starting from a reparametrization invariant form of the bosonic action for N coincident D0-branes and making it supersymmetric and κ-invariant. We shall follow the point of view of above mentioned papers that N coincident D-branes should, actually, be regarded as a single brane with non-abelian fields on its worldvolume, or a N(onabelian)D-brane. The worldvolume diffeomorphisms and κ-symmetry of the ND-brane to be conventional "abelian" symmetries in contrast to the "non-abelian" proposal of [9]. A main problem of generalizing these results to higher dimensions is to find an explicit form of the first-class constraint which is responsible for the worldline reparametrization invariance of the non-abelian ND0-brane actions. In a

JHEP02(2003)055

recent paper [10] this problem has been solved for the case of ND0-branes in $D = 3$ and a worldline reparametrization invariant bosonic ND0-brane action of codimension two has been constructed.

The purpose of this paper is to generalize the results of [5, 6] and [10] to supersymmetric N coincident D0-branes in higher dimensional superspaces. The main result we get is a generic supersymmetric and κ-invariant action for describing coincident D0-branes with non-abelian matter fields on their worldline. Particular examples to be considered are the non-abelian ND0-branes in $N = 2$, $D = 3$ superspace and in type-IIA $D = 10$ superspace. The action is demonstrated to be in agreement with the Matrix Theory limit of the ND0-brane effective action [11].

2. Reparametrization invariant bosonic ND0-brane action

For the purposes of supersymmetrization it turns out to be convenient to use the first order formulation of the ND0-brane action in a D-dimensional Minkowski space parametrized by *commuting* coordinates $x^M = (x^0, x^i)$ $(i = 1, \ldots, D-1)$ [5]

$$S = \int d\tau \,\mathrm{Tr}\left\{\frac{1}{N}p_M \dot{x}^M + p_{\varphi^i}\dot{\varphi}^i - \frac{e(\tau)}{2N}[p_M p^M + M^2(p_i, p_\varphi, \varphi^j)]\right\}, \tag{2.1}$$

where $e(\tau)$ is the Lagrange multiplier for the mass shell condition and $M(p_{\varphi^i}, p_i, \varphi^j)$ is an 'effective mass' which is an ordinary constant mass in the case of a single D0-brane, and which in the case of N coincident D0-branes depends on the $\mathrm{U}(N)$-valued canonical conjugate momenta $P_{\Phi^i} = \frac{1}{N} p_i I + p_{\varphi^i}$ of the non-abelian $\mathrm{U}(N)$ scalar fields $\Phi^i(\tau) = x^i(\tau) I + \varphi^i(\tau)$. $x^M = (x^0, x^i)$ are the center of mass space-time coordinates of the ND0-brane and $\varphi^i(\tau)$ are $\mathrm{SU}(N)$ valued worldline fields, traceless hermitian $N \times N$ matrices.

In the case of the ND0-brane in $D = 2$ the effective mass does not depend on $\varphi^i(\tau)$ and has the form [5]

$$M^2 = \left[\left(\mathrm{Tr}\sqrt{\left(\frac{1}{N}p_1 + p_\varphi\right)^2 + m^2}\right)^2 - p_1^2\right]. \tag{2.2}$$

In the above, m is the mass (or 'tension') of a single D0 brane.

In the $D = 3$ case the effective mass acquires a dependence on $\varphi^i(\tau)$ and becomes [10]

$$\begin{aligned} M^2(p_i, p_{\varphi^i}, \varphi^j) &= \left[\left(\mathrm{STr}\sqrt{(p^2_{\Phi^1} + p^2_{\Phi^2} + m^2)(1 - [\Phi^1, \Phi^2]^2)}\right)^2 - p_1^2 - p_2^2\right] \\ &= \left[\left(\mathrm{STr}\sqrt{[(p_{\Phi^i})^2 + m^2]\det Q^{ij}}\right)^2 - p_i^2\right], \end{aligned} \tag{2.3}$$

where the trace is symmetrized, $Q = 1$ for $D = 2$ and $Q^{ij} = \delta^{ij} + i[\Phi^i, \Phi^j] = \delta^{ij} + i[\phi^i, \phi^j]$ $(i, j = 1, 2, \ldots, D-1)$ for $D = 3$.

Using the explicit form of M in $D = 2, 3$, integrating (2.1) over the canonical momenta and imposing the static gauge $x^0 = \tau$ one gets the non-abelian DBI-like ND0-brane action of [1, 2, 3] in a flat background

$$S = -m \int d\tau\, \mathrm{STr} \left\{ \sqrt{\left(1 - \dot\Phi^i\, Q^{-1}_{ij}\, \dot\Phi^j\right) \det Q^{ij}} \right\} . \tag{2.4}$$

In higher dimensions the expression for the effective mass M should be a generalization of (2.3), but its explicit complete form has not been found yet because of technical problems.

However, as we will see in the next section, it turns out that the explicit form of M does not play any important role for the construction of a supersymmetric action for ND0-branes which happens to be consistent for a generic form of the effective mass.

3. Supersymmetrization

We shall now generalize the action (2.1) to be supersymmetric and κ-invariant in a D-dimensional $N = 2$ superspace whose Grassmann directions are parametrized by *anticommuting* spinor coordinates θ^α, α being a commulative index which stands for a spinorial index of a corresponding $\mathrm{Spin}(1, D-1)$ group and can also include an SO(2) R-symmetry index $I = 1, 2$ of $N = 2$ supersymmetry, when present, as in the case $N = 2$, $D = 3$. $\theta^\alpha(\tau)$ are Grassmann-odd coordinates in the superspace of the center of mass of the N coincident D0-branes.

A generic form of the supersymmetric ND0-brane action is [5, 6]

$$\begin{aligned} S = \int d\tau\, \mathrm{Tr} \Big\{ & \frac{1}{N} p_M(\dot x^M + i\bar\theta\gamma^M\dot\theta) - \frac{e(\tau)}{2N}\left[p_M p^M + M^2(p_i, p_\varphi, \varphi, \psi)\right] + \\ & + \frac{i}{N} M(p_i, p_\varphi, \varphi, \psi)\, \bar\theta\Gamma\dot\theta + p_{\varphi^i}\dot\varphi^i + i\bar\psi^A\dot\psi^A \Big\} , \end{aligned} \tag{3.1}$$

where in the 'Chern-Simons' part of the ND0-brane action Γ is a spinor matrix with the properties that $\Gamma\Gamma = I, \{\Gamma, \gamma^m\} = 0$ and whose explicit form depends on the dimension of the target superspace. For instance, in $D = 2$ θ^α (α=1,2) is a two component Majorana spinor and $\Gamma = \gamma^2$ where, $\gamma^2 = \gamma^0\,\gamma^1$ are $D = 2$ Dirac matrices. In $N = 2$, $D = 3$ we have two two-component Majorana spinors $\theta^{I\alpha}$ $(I = 1, 2)$, $\gamma^M_{I\alpha,J\beta} = \delta_{IJ}\,\gamma^M_{\alpha\beta}$ and $\Gamma_{I\alpha,J\beta} = \epsilon_{IJ}\epsilon_{\alpha\beta}$ where ϵ are 2×2 antisymmetric unit matrices. In type-IIA $D = 10$ superspace θ^α is a 32-component Majorana spinor and $\Gamma = \gamma^{11}$.

$\psi^A(\tau)$ appearing in the action (3.1) are Grassmann-odd SU(N)-valued worldline fields. Note that we assume that the effective mass of the supersymmetric ND0-brane (3.1) can also depend on the fields $\psi^A(\tau)$ (or rather on their belinears). The number of $\psi^A(\tau)$, labeled by the index A, is half the number of $\theta^\alpha(\tau)$ (in $D = 2$ $A = 1$; in $D = 3$ $A = 1, 2$ and coincides with the Majorana spinor index α, and in $D = 10$ $A = 1, \ldots, 16$). $\psi^A(\tau)$ are assumed to transform under a spinor representation of $\mathrm{SO}(D-1)$ and should be regarded (like the bosonic worldline variables $\varphi^i(\tau)$) as non-abelian counterparts of spinorial coordinates of N D0-branes gauge fixed with $N-1$ κ-symmetries, the action (3.1) being invariant under a remaining single κ-symmetry.

The dependence of the effective mass on ψ^A can be determined (at least partially) by comparing the ND0-brane action (3.1) with that of Matrix theory [11], which we perform in section 4. Thus up to the second order in ψ^A we have

$$M(p_i, p_\varphi, \varphi, \psi) = M_{bos}(p_i, p_\varphi, \varphi) + i\,\mathrm{Tr}(\psi^A \gamma^i_{AB}[\phi^i, \psi^B]) + O(\psi^4) + \cdots\,, \tag{3.2}$$

where γ^i_{AB} are $(D-1)$-dimensional gamma matrices and $M_{bos}(p_i, p_\varphi, \varphi)$ is the bosonic ND0-brane effective mass (2.3).

Before considering the form of the κ-symmetry transformations, let us present global target-space supersymmetry variations of the fields under which the action (3.1) is invariant

$$\begin{aligned}
\delta_\epsilon \theta^\alpha &= \epsilon^\alpha\,, & \delta_\epsilon x^M &= -i\bar\epsilon\gamma^M\theta - i\bar\epsilon\Gamma\theta\frac{\partial M(p_i, p_\varphi, \varphi, \psi)}{\partial p_i}\delta^M_i\,,\\
\delta_\epsilon p_M &= 0\,, & \delta_\epsilon e &= 0\,,\\
\delta_\epsilon \varphi^i &= -i\bar\epsilon\Gamma\theta\left[\frac{\partial M(p_i, p_\varphi, \varphi, \psi)}{N\partial p_{\varphi^i}}\right]_{\text{trless}}\,, & \delta_\epsilon p_{\varphi^i} &= i\bar\epsilon\Gamma\theta\left[\frac{\partial M(p_i, p_\varphi, \varphi, \psi)}{\partial \varphi^i}\right]_{\text{trless}}\,,\\
\delta_\epsilon \psi &= i\bar\epsilon\Gamma\theta\left[\frac{\partial M(p_i, p_\varphi, \varphi, \psi)}{\partial \psi}\right]_{\text{trless}}\,,
\end{aligned} \tag{3.3}$$

where $\delta^M_i = 1$ when $M = i$, and $\delta^M_i = 0$ when $M = 0$ or $M \neq i$.

Note that since the effective mass $M(p_{\varphi^i}, p_i, \varphi, \psi)$, is a function of p_i, the global supersymmetry transformation of the spatial coordinates x^i gets modified, which is the price for the model to be non-invariant under Lorentz transformations in D-dimensional space-time (see [5, 6] for a detailed discussion of this point). For similar reasons also the non-abelian SU(N) worldvolume fields $\varphi(\tau)$, $\psi(\tau)$ and the non-abelian momenta $p_\varphi(\tau)$ non-trivially transform under target-space supersymmetry.

At this point we should note that $M(p_i, p_\varphi, \varphi, \psi)$ itself is *invariant* under the supersymmetry variations (3.3) and under κ-variations (see (3.6) below) which is crucial for the action (3.1) to be supersymmetric and κ-invariant. This is the case for a generic form of $M(p_\varphi, p_i, \varphi, \psi)$, the only requirement being that M does not depend on x^M and θ^α.

In $D = 2$ the effective mass does not depend on φ (see (2.2)). If in the supersymmetric case M does not depend on ψ the supersymmetric ND0-brane action in $D = 2$ possesses redundant local worldvolume supersymmetries under the variations $\delta\varphi = 2\alpha(\tau)\psi, \delta\psi = \alpha(\tau)p_\varphi$ with the anticommuting parameter $\alpha(\tau)$ [5]. In higher space-time dimensions M acquires the dependence on φ (2.3), and the redundant local worldvolume supersymmetries disappear, even if M does not depend on ψ, as one can directly verify.

The supersymmetry algebra of the transformations (3.3) generated by the Poisson brackets of the Noether supercharges derived from the action (3.1) has the following form

$$\{Q_\alpha, Q_\beta\} = 2ip_M\gamma^M_{\alpha\beta} + 2iM(p_i, p_\varphi, \varphi, \psi)\,\Gamma_{\alpha\beta}\,, \tag{3.4}$$

where

$$Q_\alpha = \pi_\alpha + ip_M(\gamma^M\theta)_\alpha + iM(p_i, p_{\varphi^i}, \varphi, \psi)\,(\Gamma\theta)_\alpha \tag{3.5}$$

are supercharges and π_α are the momenta conjugate to θ^α. (For the cases of $D = 2, 3$ and 10 the explicit form of γ^M and Γ has been presented below eq. (3.1)).

We observe that the superalgebra (3.4) has the "central charge" term proportional to the effective mass M which arises because the spatial coordinates x^i, the SU(N) adjoint scalars, their momenta and the SU(N) fermions nontrivially transform under supersymmetry.

We now present the κ-symmetry variations of the fields of the model under which the action (3.1) is invariant

$$\begin{aligned}
\delta_\kappa\theta &= \left(p_M\,\gamma^M + M(p_i,p_\varphi,\varphi,\psi)\,\Gamma\right)\,\kappa(\tau)\,, & \delta_\kappa x^M &= i\delta_\kappa\bar\theta\gamma^M\theta + i\delta_\kappa\bar\theta\Gamma\theta\,\frac{\partial M}{\partial p_i}\,\delta_i^M\,,\\
\delta_\kappa\,p_M &= 0\,, & \delta_\kappa e &= 4i\kappa^\alpha\dot\theta_\alpha\,,\\
\delta_\kappa\varphi^i &= i\delta_\kappa\bar\theta\Gamma\theta\,\left[\frac{\partial M(p_i,p_\varphi,\varphi,\psi)}{N\partial p_{\varphi^i}}\right]_{\rm trless}\,, & \delta_\kappa p_{\varphi^i} &= -i\delta_\kappa\bar\theta\Gamma\theta\,\left[\frac{\partial M(p_i,p_\varphi,\varphi,\psi)}{\partial\varphi^i}\right]_{\rm trless}\\
\delta_\kappa\psi &= -i\delta_\kappa\bar\theta\Gamma\theta\,\left[\frac{\partial M(p_i,p_\varphi,\varphi,\psi)}{\partial\psi}\right]_{\rm trless}\,,
\end{aligned}\tag{3.6}$$

where $\kappa^\alpha(\tau)$ is the local fermionic parameter. Let us stress again that the effective mass is invariant under the κ-symmetry variations as well as under the target space supersymmetry.

4. Comparison with Matrix Theory

In an certain limit of type-IIA string theory the non-abelian DBI action for ND0-branes is known to reduce to the Matrix theory action [11]

$$S = m\,{\rm Tr}\int d\tau\left(\frac{1}{2}\,\dot\Phi^i\dot\Phi^j - \frac{1}{4}[\Phi^i,\,\Phi^j]\,[\Phi^i,\,\Phi^j] + i\Theta^A\dot\Theta^A - i\Theta^A\gamma^i_{AB}[\Phi^i,\,\Theta^B]\right),\tag{4.1}$$

where $\Phi^i(\tau) = x^i\,I + \phi^i$ are the ND0-brane U(N)-valued scalar fields and Θ^A are U(N)-valued fields transforming under a spinor representation of SO$(D-1)$.

To reduce the ND0-brane action (3.1) to the Matrix theory action (4.1) and to relate θ^α and ψ^A to Θ^A we should fix the worldvolume diffeomorphisms and the κ symmetry by imposing a static gauge

$$x^0 = \tau\,,\quad \theta^2_\alpha = 0 \quad \text{in } D=3 \qquad \text{and} \qquad \theta^\alpha = (\gamma^{11}\theta)^\alpha \quad \text{in } D = 2,10 \tag{4.2}$$

so that, for example in $D = 10$, 32-component θ^α reduces to a 16-component Majorana-Weyl spinor θ^A which together with ψ^A form the U(N) spinor $\Theta^A = \theta^A\,I + \psi^A$.

Then we keep in the action (3.1) the terms up to the second order in the momenta p_{Φ^i} and in $[\Phi^i,\,\Phi^j]$, integrate over p_0 and p_{Φ^i} and skip all derivative terms except for the kinetic terms. We thus arrive at the matrix theory action (4.1). Note that the form of the last term in (4.1) has prompted us the dependence of the effective mass (3.2) on the non-abelian fermions ψ^A.

5. The Lorentz-covariant super-ND0-brane system

As it has been considered in detail in [5, 6] the ND-brane actions proposed in [1, 2, 3] are not Lorentz (or diffeomorphism) invariant in D-dimensional target space except for the space filling branes. This is not because they are constructed in the static gauge, which can be removed by restoring the reparametrization invariance in the way discussed above. The main reason is that the center-of-mass U(1) coordinates x^i of the ND-brane get mixed with the non-abelian SU(N) scalar fields φ^i, which follows from the form of the action (2.4).

This implies that the motion of the center of mass of the ND-brane as a whole depends on the internal excitations inside the system, which is rather strange. One may wonder whether the computation of string amplitudes associated with N coincident D-branes of higher codimension confirms such an effect.

We now consider the Lorentz-covariant counterpart of the above model. For this we assume the effective mass in (3.1) to be independent of the spatial momenta p_i. This ensures the free motion of the ND0-brane center of mass. If so, we can now assume that the indices i and A of $\varphi^i(\tau)$, p_{φ^i} and ψ^A are the indices of a corresponding representation of an *independet* internal group $\mathrm{SO}(D-1)$ which *a priori* is not related to the spatial subgroup of the Lorentz group $\mathrm{SO}(1, D-1)$.

The supersymmetric and κ invariant action for the Lorentz invariant ND0-brane is

$$S = \int d\tau\, \mathrm{Tr}\Big\{ \frac{1}{N} p_M (\dot{x}^M + i\theta\gamma^M \dot{\theta}) - \frac{e(\tau)}{2N} \left[p_M p^M + M^2(p_\varphi, \varphi, \psi) \right] + \\ + \frac{i}{N} M(p_\varphi, \varphi, \psi)\, \bar{\theta}\Gamma\dot{\theta} + p_\varphi \dot{\varphi} + i\bar{\psi}\dot{\psi} \Big\}. \tag{5.1}$$

The variation properties of the fields with respect to the symmetries of the action (5.1) are the same as written in equations (3.3), (3.6), except that now the spatial coordinates x^i transform in the standard way under the κ-symmetry and target-space supersymmetry, i.e. there is no contribution of derivatives of the effective mass into their variation, and the variation of $x^M = (x^0, x^i)$ is Lorentz invariant.

6. Conclusion and discussion

We have constructed the action for N coincident D0-branes which is target-space supersymmtric and invariant under local worldline fermionic κ-symmetry in a D-dimensional $N = 2$ superspace, particular cases being $D = 2, 3$ and type-IIA $D = 10$. The action has a generic structure determined by a super- and κ-invariant effective mass which is a generic function of the non-abelian $\mathrm{SU}(N)$ fields of the model and their momenta, and which also depends on the spatial momentum of the center of mass of the system. It is crucial for the invariance of the action that the effective mass does not depend on x^M and θ^α. In the bosonic limit the explicit form of the effective mass is dictated by the non-abelian DBI structure of the ND0-brane action, and it should still to be determined for $D = 10$.

We have compared the supersymmetric ND0-brane action with that of Matrix theory, which has allowed us to fix the dependence of the effective mass (3.2) on the non-abelian $\mathrm{SU}(N)$ fermions ψ^A up to the second order. A problem which remains is to determine higher order fermionic terms in the effective mass.

It would be of interest to apply the T-duality procedure to the supersymmetric ND0-brane model for getting supersymmetric actions for higher dimensional coincident D-branes.

It would be also interesting to compare the above ND0-brane construction with that of [7] for the space filling supersymmetric ND2-brane by performing the worldvolume dimensional reduction of the latter.

JHEP02(2003)055

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